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Scalar effective potential for D7 brane probes which break chiral symmetry

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Abstract

We consider D7 brane probes embedded in deformed $AdS_5 \times S^5$ supergravity backgrounds which are non-supersymmetric in the interior. In the context of the generalised AdS/CFT correspondence, these setups are dual to QCD-like theories with fundamental matter which display chiral symmetry breaking by a quark condensate. Evaluating the D7 action for a surface instanton configuration gives rise to an effective potential for the scalar Higgs vev in the dual field theory. We calculate this potential for two specific supergravity backgrounds. For a metric due to Constable and Myers, we find that the potential is asymptotically bounded by a $1/Q^4$ behaviour and has a minimum at zero vev. For the Yang-Mills* background we find that the Higgs potential scales quadratically with the Higgs vev. This corresponds to a canonical mass term and the embedding is again stable.

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1 Introduction

D7 brane probes have proved a versatile tool for including quark fields into the AdS/CFT correspondence [1]. Strings stretching between the D7s and the D3 branes of the original AdS/CFT construction provide $\mathcal{N}=2$ fundamental hypermultiplets [2]. Karch and Katz [3] proposed that the open string sector on the world-volume of a probe D7 brane is holographically dual to quark–anti-quark bilinears $\bar{\psi}\psi$. There have been many studies using probe D7s in a variety of gravity backgrounds [4, 5]. In this way a number of non-supersymmetric geometries have been shown to induce chiral symmetry breaking [6, 7] (related analyses are [8]), with the symmetry breaking geometrically displayed by the D7 brane's bending to break an explicit symmetry of the space. Meson spectra are also calculable [9].

In scenarios involving two or more D7 probes, the Higgs branch spanned by squark vevs $\langle \bar{q}q \rangle$ can be identified with instanton configurations on the D7 world-volume [10, 11]. These configurations are the standard four-dimensional instanton solutions living in the four directions of the D7 world-volume transverse to the D3 branes. The scalar Higgs vev in the field theory is identified with the instanton size on the supergravity side [1]. In the case of a probe in AdS space, there is a moduli space for the magnitude of the instanton size or the scalar vev. In less supersymmetric gravity backgrounds though, the moduli space is expected to be lifted. A potential may be generated, which either may have a stable vacuum selecting a particular scalar vev, or may have a run-away behaviour. In a previous paper [12], we analyzed the Higgs branch of the $\mathcal{N}=4$ gauge theory at finite temperature and density. For the finite temperature case we found a stable minimum for the squark vev which undergoes a first order phase transition as a function of the temperature (or equivalently of the quark mass). On the other hand, in the presence of a chemical potential the squark potential leads to an instability, indicating Bose-Einstein condensation. This implies also for other supergravity backgrounds that the squark vev may potentially be large, in which case physical observables such as meson masses may be significantly modified.

In this paper we use the method developed in [12] to analyze the scalar potential in the case of two quark flavours in two gravity backgrounds describing gauge configurations that induce chiral symmetry breaking. The first is the Constable-Myers dilaton flow geometry [13](see also [14]). This geometry was the first used to display chiral symmetry breaking in [6], where it was used for its simplicity. The dilaton flow retains an unspoilt S^5 which makes the probe analysis particularly simple. From the field theory point of view, this background has the essential property that it breaks supersymmetry completely, such that the strong gauge dynamics may generate a quark

[1]Strictly speaking, it is a mixed Coulomb-Higgs branch which enters this duality, for details see [11].

bilinear, which would be forbidden if some of the supersymmetry were preserved.

The Constable-Myers geometry corresponds to the addition of a non-zero vev for the operator TrF^2 to the $\mathcal{N}=4$ gauge theory. This is expected to be unstable and therefore the supergravity background is also expected to display instabilities. Here however we show that the scalar quark potential is well behaved and drives the vev to zero [2]. Asymptotically the potential has the form of a constant minus a $1/Q^4$ term, with $Q^2 = \langle \bar{q} q \rangle$. This behaviour is determined essentially by dimensional counting since the supersymmetry breaking parameter, $\mathrm{Tr}F^2$ is dimension four.

The Yang Mills* geometry [15] is an example of a more realistic dual. An equal mass term and/or condensate is introduced for each of the four gauginos of the $\mathcal{N}=4$ theory breaking supersymmetry. Scalar masses are then generated radiatively leaving a dual of pure Yang Mills theory in the infra-red. The gaugino mass terms form an operator in the **10** representation of SO(6), such that when the dual supergravity scalar is switched on the S^5 of the geometry becomes crushed (to two S^2). The embedding of a D7 brane in this geometry is a complicated multi-coordinate problem and the full embedding is not known. Nevertheless, in [16] it was shown that the core of the geometry is repulsive to a D7 at all but one isolated point in the parameter space of gaugino mass/condensate, which is the expected signal for chiral symmetry breaking. Here we show that for large Higgs vev, the effective potential for the squark vev is quadratic and positive, indicating that small squark vevs are favoured for the Yang-Mills* background. The UV of the theory therefore has a standard scalar quark mass term. This is possible because the supersymmetry breaking term, the gaugino mass is dimension one.

2 The Basic $N_f = 2$ Squark Moduli Space

Consider a probe of two coincident D7 branes in $\mathrm{AdS}_5 \times S^5$. This corresponds to two fundamental hypermultiplets in the dual gauge theory. The metric of $AdS_5 \times S^5$ is given by

$$ds^2 = \frac{u^2}{R^2}\, dx_{//}^2 + \frac{R^2}{u^2}\left(d\rho^2 + \rho^2 d\Omega_3^2 + du_5^2 + du_6^2\right), \tag{1}$$

where we have written the four coordinates on the D7 world-volume in spherical coordinates ($\rho^2 = u_1^2 + u_2^2 + u_3^2 + u_4^2$). u_5 and u_6 are the directions transverse to the D7 and $u^2 = \rho^2 + u_5^2 + u_6^2$. R is the AdS radius with $R^4 = 4\pi g_s^2 N \alpha'^2$. The dilaton and the four-form potential read

$$e^\Phi = g_s, \qquad C_{(4)} = \frac{u^4}{g_s\, R^4}\, dx^0 \wedge dx^1 \wedge dx^2 \wedge dx^3. \tag{2}$$

[2] Note in the first preprint version of this paper we had mislaid a factor of the dilaton which mistakenly suggested a run-away potential.

With G_{ab} the pullback of the metric (1), the *Einstein frame* action for the D7 probe is given by

$$S_{D7} = -\frac{T_7}{g_s^2} \int d^4x \; d\rho \; d\Omega_3 \, \rho^3 e^{\Phi} \, \mathrm{Tr} \sqrt{- \det \left(G_{ab} + 2\pi\alpha' e^{-\frac{\Phi}{2}} F_{ab} \right)} \; + T_7 \int C_{(4)} \wedge \mathrm{Tr} \, e^{2\pi\alpha' F}. \quad (3)$$

The regular solutions of the resulting equations of motion for the embedding of the D7 are simply $u_5, u_6 = const.$ (the quark mass is $m^2 = u_5^2 + u_6^2$).
Next consider SU(2) instantonic solutions for the two-form F_{ab} in (3), with the standard form [3]

$$A_4 = \frac{i}{g} \frac{u_j \sigma_j}{Q^2 + \rho^2}, \qquad A_j = -\frac{i}{g} \frac{u_4 \sigma_j + \epsilon_{jkl} u_k \sigma_l}{Q^2 + \rho^2}, \qquad j = 1, 2, 3, \quad (4)$$

where the σ_i are the usual Pauli matrices, and we sum over repeated indices. Q is the instanton size, which is identified with the vev of the squark fields in the dual gauge theory. For details see [11]. The instantonic configuration is self-dual with respect to a flat metric, and gives

$$\mathrm{Tr} \, F_{mn} F_{mn} = -\frac{96}{g^2} \frac{Q^4}{(Q^2 + \rho^2)^4}, \qquad m, n = u_1, \ldots, u_4. \quad (5)$$

Expanding (3) to second order in F and dropping the leading constant, we obtain

$$S_{D7} = -T_7 \, \frac{(2\pi^2 \alpha')^2}{2 \, g_s R^4} \int d^4x \; d\rho \; \rho^3 (\rho^2 + m^2)^2 \left(\mathrm{Tr} \, F_{ab} F_{ab} + \mathrm{Tr} \, F^*_{ab} F_{ab} \right) . \quad (6)$$

Since for any instanton $F + F^* = 0$, the two terms in (6) cancel exactly and we are left with the expected moduli space of the $\mathcal{N} = 2$ theory [4].

3 Constable-Myers geometry

The dilaton-flow geometry of Constable and Myers [13] is asymptotically AdS at large radius, but is deformed in the interior of the space by an R-chargeless parameter of dimension four (b^4 in what follows). This geometry is interpreted as being dual to $\mathcal{N} = 4$ gauge theory with a non-zero expectation value for $\mathrm{Tr} \, F^2$. It was used in [6] to study chiral symmetry breaking because of its particularly simple form with a flat six-dimensional plane transverse to the D3 branes. The core of the geometry is singular[5]. D7 brane probes in the geometry are repelled by the central singularity giving rise to chiral symmetry breaking.

[3]For our purposes it is enough to focus on the slice of the Higgs branch corresponding to a single, radially symmetric instanton.

[4]Note there is a moduli space even in the presence of a quark mass because the adjoint scalar vev can be set to effectively remove that mass.

[5]This singularity may presumably be lifted by the D3 branes forming some sort of fuzzy sphere in the interior of the space.

In *Einstein frame*, the Constable-Myers geometry is given by

$$ds^2 = H^{-1/2} K^{\delta/4} dx_4^2 + H^{1/2} K^{(2-\delta)/4} \frac{u^4 - b^4}{u^4} \sum_{i=1}^{6} du_i^2, \tag{7}$$

where

$$K = \left(\frac{u^4 + b^4}{u^4 - b^4}\right), \qquad H = K^\delta - 1, \qquad \delta = \frac{R^4}{2b^4}, \qquad \Delta^2 = 10 - \delta^2 .$$

b is the deformation parameter. The dilaton and four-form are

$$e^{2\Phi} = g_s^2 K^\Delta, \qquad C_{(4)} = (g_s\, H)^{-1}\, dt \wedge dx \wedge dy \wedge dz. \tag{8}$$

We now place two D7 brane probes into the geometry, such that they fill the x_4 directions and four of the u directions (we write $\sum_{i=1}^4 u_i^2 = \rho^2$). The equation of motion that determines how the D7 lies in the u_5 direction as a function of ρ is, with $u_6 = 0$,

$$\frac{d}{d\rho}\left[\frac{e^\Phi \mathcal{G}(\rho, u_5)}{\sqrt{1 + (\partial_\rho u_5)^2}}(\partial_\rho u_5)\right] - \sqrt{1 + (\partial_\rho u_5)^2}\frac{d}{d\bar{u_5}}\left[e^\Phi \mathcal{G}(\rho, u_5)\right] = 0, \tag{9}$$

$$\mathcal{G}(\rho, u_5) = \rho^3 \frac{((\rho^2 + u_5^2)^2 + b^4)((\rho^2 + u_5^2)^2 - b^4)}{(\rho^2 + u_5^2)^4}.$$

Asymptotically the solutions take the form $u_5 = m + c/\rho^2$. The regular solutions obtained numerically are shown in Figure 1a). There is a U(1) symmetry in the $u_5 - u_6$ plane which corresponds to the axial symmetry of the quarks. For $m = 0$ the solution preserves this symmetry asymptotically, since $u_5(\rho)$ goes to zero for large ρ. On the other hand, the symmetry is broken in the interior, where the solution u_5 is non-zero. This is the signal of chiral symmetry breaking, the D7 probe being repelled by the singularity in the interior.

Now we consider the action of an instanton in the $u_1 - u_4$ directions on the D7 world-volume with the above embeddings. In the field theory this corresponds to computing the potential energy in the non-supersymmetric theory with the scalar vev from the supersymmetric theory's moduli space. The induced metric on the embedded branes reads

$$G_{\mu\nu} = H^{-\frac{1}{2}} K^{\frac{\delta}{4}} \eta_{\mu\nu}, \qquad G_{u_i u_j} = H^{\frac{1}{2}} K^{\frac{2-\delta}{4}} \frac{u^4 - b^4}{u^4}\left(\delta_{ij} + u_i u_j \frac{(\partial_\rho u_5)^2}{\rho^2}\right) \quad i,j = 1, \ldots, 4, \tag{10}$$

$$\det G = K^2 \frac{(u^4 - b^4)^4}{u^{16}}\left(1 + (\partial_\rho u_5)^2\right) .$$

We evaluate the D7-brane action on the space of fields strengths which are self-dual with respect to the transverse part of the metric. With the coordinate transformation $\tilde{u}^i = J(\rho)\, u^i$, the induced metric becomes a (conformally) flat metric provided that the function J satisfies

$$\rho^2\, (\partial_\rho J)^2 + 2\, \rho\, J\, \partial_\rho J - J^2\, \left(\partial_\rho u_5(\rho)\right)^2 = 0\,, \tag{11}$$

together with the boundary condition $J(\rho \to \infty) = 1$. In the new coordinates $\tilde{u}^m$, the single instanton centered at the origin has the usual form (4), and the action is given by (5).

In the new set of coordinates we have, at second order in α',

$$S_{D7} = -T_7 \, \frac{6(4\pi\alpha')^2}{g_s \, g^2} \int d^4x \int d^4\tilde{u} \left[\sqrt{-\det G} \; G_{\tilde{u}\tilde{u}}^{-2} - \, g_s \, C_{(4)}\right] \frac{Q^4}{(\tilde{\rho}^2 + Q^2)^4} \, . \tag{12}$$

In the original coordinates

$$S_{D7} = -T_7 \, \frac{6(4\pi\alpha')^2}{g_s \, g^2} \int d^4x \int d^4u \, \left[\sqrt{-\det G} \; G_{uu}^{-2} - {g_s}^2 \, C_{(4)}\right] \frac{Q^4 J(\rho)^4}{(J(\rho)^2\rho^2 + Q^2)^4} \left(1 + \frac{\rho \, J'(\rho)}{J(\rho)}\right) .$$

Since the instanton configuration is static, the integral over the four u coordinates coincides with $-V$, the potential for the field Q. In polar coordinates we have

$$\begin{aligned} V(Q) &= T_7 \, \frac{3(8\pi^2\alpha')^2}{g_s \, g^2} \int d\rho \, \rho^3 \left[\frac{Q^4 J(\rho)^4}{(J(\rho)^2\rho^2 + Q^2)^4}\right] \frac{Z^{\frac{\delta}{2}} - 1}{Z^{\delta} - 1} \left(1 + \rho \, \frac{J'(\rho)}{J(\rho)}\right) \qquad (13) \\ Z &= \frac{(\rho^2 + u_5(\rho)^2)^2 + b^4}{(\rho^2 + u_5(\rho)^2)^2 - b^4} \, . \end{aligned}$$

We plot this potential for different values of the quark mass m_q in figure 1b. For all m_q the potential has a stable minimum at zero vev.

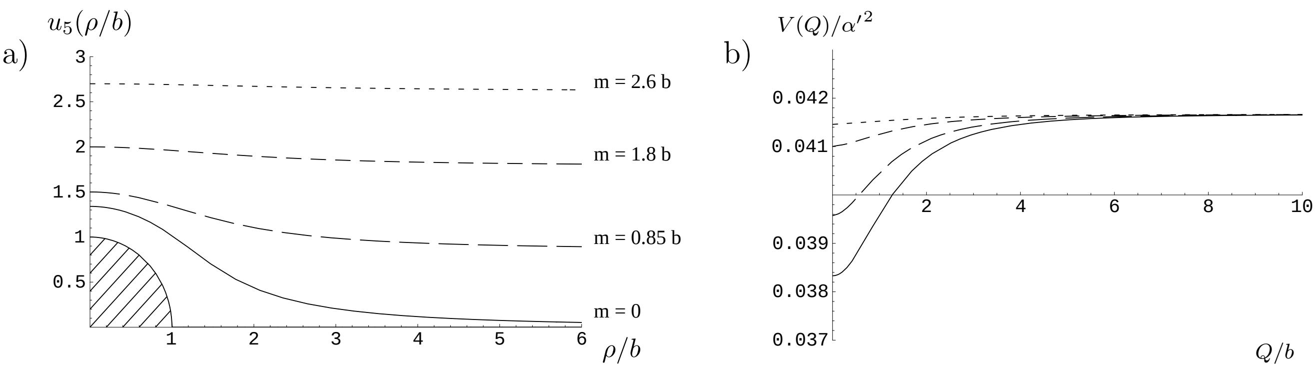

Figure 1: a) Plot of the D7 brane embedding in the Constable-Myers geometry for different values for m_q. b) Potential versus the Higgs vev Q for the values of m_q shown in 1a). All distances are expressed in units of b, while the potential is proportional to α'^2.

Both the IR and the UV behaviour of this solution may be checked analytically.
Let us first consider the small Q case. It is always possible to split the ρ integration of (13) in the integration over the two intervals $\rho \leq Q$ (or $\rho \in [0, Q]$) and $Q < \rho$ (or $\rho \in (Q, \infty)$).
At small ρ, both the embedding u_5 (see Fig. 1a) and the function J go to a constant plus corrections of order ρ^2,

$$u_5 = u_5(0) + \frac{2b^8 - \Delta \, b^4 m^4}{2m(m^8 - b^8)} \, \rho^2 + \ldots , \qquad\qquad J = J(0) + J(0) \frac{(2b^8 - \Delta \, b^4 m^4)^2}{4m^2(m^8 - b^8)^2} \, \rho^2 + \ldots .$$

Then we have for the first interval

$$V_{[0,Q]} \;=\; c_0 \int_0^Q d\rho\, \rho^3 \,\frac{Q^4 J(0)^4}{(J(0)^2\rho^2+Q^2)^4}\,(c_1+c_2\,\rho^2+\ldots), \tag{14}$$

where

$$c_0 = \frac{3T_7(8\pi^2\alpha')^2}{g_s\, g^2},$$

and, for example,

$$c_1 = \left(1+\left(\frac{u_5(0)^4+b^4}{u_5(0)^4-b^4}\right)^{\frac{\delta}{2}}\right)^{-1}.$$

With the substitution $\tilde{\rho}=\rho/Q$ this integral is easily evaluated and gives

$$V_{[0,Q]} = \frac{c_0\, c_1}{24} + \frac{c_0\, c_2}{48}\, Q^2 + \ldots\;.$$

In the second interval where $\rho > Q$ we may neglect the Q contribution to the denominator of the integrand. For Q small we have

$$V_{(Q,\infty)} \;=\; c_0\, Q^4 \int d\rho\; f(\rho) \;\sim\; c_0\,\mathcal{I}\,Q^4\,, \tag{15}$$

with $\mathcal{I}$ a numerical constant. Thus, summing up the two integration intervals, we recover the stable minimum for V around $Q=0$.

The large Q case is analogous. Even without splitting the integration interval, we may look at large Q solutions where the action will be dominated at large ρ. For large Q and ρ the action has the expansion

$$V = \frac{3T_7(8\pi^2\alpha')^2}{g_s\, g^2}\int \rho^3 d\rho\, \frac{Q^4}{(\rho^2+Q^2)^4}\left[\frac{1}{2}-\frac{\delta\, b^4}{4\, u^4}+\frac{1}{u^6}\frac{10\,c^2+3\,\delta\, b^4\, m^2}{6}\;+\;\ldots\right]. \tag{16}$$

The asymptotic solution for the embedding has $u^2=\rho^2+u_5^2=\rho^2+m^2+2mc/\rho^2+\ldots$. Finally we rescale ρ by Q to make the integrals dimensionless. This gives

$$V = \frac{3T_7(8\pi^2\alpha')^2}{g_s\, g^2}\left[\frac{1}{2}\mathcal{I}_1 - \frac{b^4}{Q^4}\frac{\delta}{4}\mathcal{I}_2 + \frac{1}{Q^6}\frac{10\,c^2+3\,\delta\, b^4\, m^2}{6}\,\mathcal{I}_3\;+\;\ldots\right], \tag{17}$$

where the integrals, $\mathcal{I}_n$, are easily extracted from (16).

It is interesting to use the same procedure of splitting the integration interval also in the large Q case. For $\rho\to\infty$ again both u_5 and J go to a constant (m and 1 respectively), such that

$$V_{(Q,\infty)} = \frac{c_0}{2}\int_Q^\infty d\rho\, \rho^3\, \frac{Q^4}{(\rho^2+Q^2)^4}\,\left(1+O(\rho^{-2})\right). \tag{18}$$

With the substitution $\tilde{\rho}=\rho/Q$ we have

$$V_{(Q,\infty)} = \frac{c_0}{2}\int_1^\infty d\tilde{\rho}\, \frac{\tilde{\rho}^3}{(\tilde{\rho}^2+1)^4}\,\left(1+O(\tilde{\rho}^{-2})\right) \;=\; \frac{c_0}{48}+\ldots. \tag{19}$$

Thus we see that the constant term of (17) originates from the instanton configuration of size Q probing the large ρ region of the background, as it is natural since that region is asymptotically AdS where a flat potential as to be expected. For the interval $[0, Q]$ we can instead neglect the ρ^2 term in the denominator of the integrand and we are left, for Q large enough, with

$$V_{[0,Q]} \;=\; c_0 \int d\rho\, \rho^3\, \frac{Q^4 J(\rho)^4}{Q^8} \frac{Z^{\frac{\delta}{2}}-1}{Z^\delta - 1} \;=\; \frac{c_0}{Q^4} \int d\rho\, f(\rho) \;\sim\; c_0\, \mathcal{I}'\, \frac{1}{Q^4}\,. \tag{20}$$

We see that the deviation from the flat potential of the AdS case originates from the instantonic configuration, even of very large size, probing the interior of the space. Of course summing up the results for the two integration intervals we recover the first two terms of (17).

We note that the expression (17) vanishes for $b \to 0$ as expected. The leading term is a constant independent of Q, which corresponds to the fact that the metric returns to $AdS_5 \times S^5$ for small b. This constant is also seen in the numerical result shown in figure 1b). Moreover, the expansion (17) successfully reproduces the form of the potential with a $-b^4/Q^4$ term that forces the vev in towards zero. In fact, this behaviour is expected from dimensional analysis since the deformation has dimension four and the non-constant part of the potential must vanish as $b^4 \to 0$. This term with negative powers of the quantity associated to the squark bilinear operator indicates that the field theory has a rather peculiar UV lagrangian with self interactions involving inverses of the fields. This is probably the result of integrating out the far UV completion, that is the $\mathcal{N} = 4$ massive degrees of freedom.

We see that the screening effect which keeps the D7 embedding away from the singularity region also protects the open string sector modes associated to the squark vev from the instabilities which may arise in the closed string sector due to the singular closed string background.

Let us provide some intuition for why the brane configuration disfavours large instantons. We suggest the essential reason is that the background metric causes volume elements to expand as $u = b$ is approached: The D7 brane bends away from the origin in order to minimize its world-volume. The natural expectation is that the instanton action for small instantons around $\rho = 0$ will grow with the size of the instanton preferring zero size instantons. The same argument breaks down in pure AdS because the four-form term conspires to cancel the $\sqrt{\det G}$ volume term. When supersymmetry is broken, this cancellation no longer works and the increase in the volume term is the stronger effect. This ensures a stable minimum for the Higgs vev.

4 Yang-Mills* geometry

Next we consider the Yang-Mills* geometry [15]. This is a deformation of AdS by a supergravity field λ in the **10** of SO(6) and corresponds to an equal mass and/or condensate for each of the four adjoint fermions of the $\mathcal{N}=4$ gauge theory. The adjoint scalar fields also acquire a mass. This geometry therefore naturally describes a pure Yang-Mills theory in the infrared.

The geometry is complicated with the S^5 crushed to two S^2. This implies that the D7 brane embedding has a complicated angular dependence, making it hard to find the full D7 embedding. Therefore we restrict ourselves to an asymptotic analysis, and check that the scalar potential for the D7 probes is bounded.

4.1 The Background

The full 10d geometry reads

$$ds_{10}^2 = (\xi_+\xi_-)^{\frac{1}{4}} ds_{1,4}^2 + (\xi_+\xi_-)^{-\frac{3}{4}} R^2\, ds_5^2 \tag{21}$$

$$ds_{1,4}^2 = e^{2A(r)}\eta_{\mu\nu}dx^\mu dx^\nu + R^2 dr^2, \qquad ds_5^2 = \xi_- \cos^2\alpha\ d\Omega_+^2 + \xi_+ \sin^2\alpha\ d\Omega_-^2 + \xi_+\xi_- d\alpha^2$$

with $d\Omega_\pm = d\theta_\pm^2 + \sin^2\theta_\pm d\phi_\pm^2$. The $\xi_\pm$ are given by

$$\xi_\pm = (\cosh\lambda)^2 \pm (\sinh\lambda)^2 \cos 2\alpha,$$

The axion-dilaton field is purely complex so the θ angle is zero

$$C + ie^{-\Phi} = \frac{i}{g_s}\sqrt{\frac{\xi_-}{\xi_+}} \tag{22}$$

The two-form potential is given by

$$A_{(2)} = iR^2 \frac{\sinh 2\lambda}{\xi_+} \cos^3\alpha\ \sin\theta_+ d\theta_+ \wedge d\phi_+ - R^2 \frac{\sinh 2\lambda}{\xi_-} \sin^3\alpha\ \sin\theta_- d\theta_- \wedge d\phi_- \tag{23}$$

Finally the four-form potential lifts to

$$F_{(5)} = F + \star F, \quad F = dx^0\wedge dx^1\wedge dx^2\wedge dx^3\wedge d\omega, \quad \omega(r) = {g_s}^{-1} R^4 e^{4A(r)} A'(r)$$

The fields λ has a potential $V = -\frac{3}{2}(1+\cosh^2\lambda)$ and, with A satisfies

$$\lambda'' + 4A'\lambda' = \frac{\partial V}{\partial\lambda}, \qquad -3A'' - 6A'^2 = \lambda'^2 + 2V \tag{24}$$

Asymptotically, where the geometry returns to AdS, the solution we are interested in behaves as

$$\begin{aligned} \lambda &= m\,\frac{e^{-r}}{R} + \left(c + \frac{1}{6}\,m^3(1+2\,r)\right)\,\frac{e^{-3r}}{R^3} + O(e^{-5r}R^{-5}); \\ A &= r - \frac{1}{6}\,m^2\,\frac{e^{-2r}}{R^2} + \left(-\frac{1}{4}\,c\,m - \frac{1}{18}\,m^4\left(1+\frac{3}{2}r\right)\right)\,\frac{e^{-4r}}{R^4} + O(e^{-6r}R^{-6}) \end{aligned} \tag{25}$$

m corresponds to an adjoint fermion mass term and c to a condensate. Numerically solving the equations of motion shows that the geometry is singular in the interior, presumably reflecting the presence of a fuzzy D3 configuration. However here we just look at the UV behaviour where the geometry is dominated by the fermion mass. This is straightforward using (25).

4.2 Asymptotic Potential

We now embed a probe of two coincident D7 branes into the Yang-Mills* geometry. The full solution has dependence on α as well as on r, the latter being known only numerically. Therefore we restrict to the asymptotic region of large r, where the embedding is known analytically. Moreover we choose to place the D7 probe in the directions given by $x_{//}$, r, α and Ω_+. The choice of Ω_+ rather than Ω_- is motivated by the fact that Ω_+ supports the NS two-form, while Ω_- supports the Ramond two-form. In this way we ensure that we consider electrically instead of magnetically charged quarks.

For large r, the embedding is determined by

$$\sin\theta_- = M\frac{e^{-r}}{R\sin\alpha} + \ldots\,, \qquad \phi_- = 0\,. \tag{26}$$

Here M is the mass of the fundamental fermion [6].

The full D7 action is

$$S^E_{D7} = -\frac{T_7}{g_s^2}\int d^8\xi\ \mathrm{Tr}\left[e^{\Phi}\sqrt{-\det\left(G_{ab} + e^{-\frac{\Phi}{2}}B_{ab} + 2\pi\alpha' e^{-\frac{\Phi}{2}}F_{ab}\right)}\right] + T_7\int C_{(4)}\wedge \mathrm{Tr}\ e^{2\pi\alpha' F} \tag{27}$$

with $d^8\xi$ expressed in term of the embedding coordinates $d^4x\,dr\,d\alpha\,d\theta_+ d\phi_+$. G_{ab} and B_{ab} are the pullbacks of the metric and of the B field respectively.

At lowest order in α' we find a term independent of the instanton - and hence of the scalar

[6] In pure AdS we put the D7 along $u_5 = Re^r\sin\alpha\sin\theta_- = M$, $\quad u_6 = Re^r\sin\alpha\sin\theta_-\sin\phi_- = 0$, hence the solution (26) in the new set of coordinates.

vev Q - which we drop. The linear order in α' traces to zero, while at second order we have

$$
\begin{aligned}
S_{D7} &= S_{DBI} + S_{WZ}\,, \\
S_{DBI} &= -T_7\,\frac{2\,\pi^2\alpha'^2}{g_s^2}\int d^8\xi\;\left\{\sqrt{\frac{-\det G}{(1+e^{-\Phi}B_{\theta_+\phi_+}B^{\theta_+\phi_+})(1+g_{\theta_-\theta_-}(\partial_r\theta_-\partial^r\theta_- + \partial_\alpha\theta_-\partial^\alpha\theta_-))}}\right. \\
&\quad \mathrm{Tr}\Bigg[\sum_{\substack{a\neq b;\\ a,b\in(r,\alpha,\theta_+,\phi_+)}} F_{ab}F^{ab} \;+\; g_{\theta_-\theta_-}\sum_{\substack{a\neq b\neq c;\\ a\in(r,\alpha);\\ b,c\in(r,\alpha,\theta_+,\phi_+)}}\partial_a\theta_-\partial^a\theta_- F_{bc}F^{bc} \;-\; 2\,g_{\theta_-\theta_-}\sum_{a\in(\theta_+,\phi_+)}\partial_r\theta_-\partial_\alpha\theta_- F^{ra}F^{\alpha}_{a} \\
&\quad \left. + e^{-\Phi}B_{\theta_+\phi_+}B^{\theta_+\phi_+}\left(F_{r\alpha}F^{r\alpha} - F_{\theta_+\phi_+}F^{\theta_+\phi_+}\frac{(1+g_{\theta_-\theta_-}(\partial_r\theta_-\partial^r\theta_- + \partial_\alpha\theta_-\partial^\alpha\theta_-))}{(1+e^{-\Phi}B_{\theta_+\phi_+}B^{\theta_+\phi_+})}\right)\Bigg]\right\}, \\
S_{WZ} &= -T_7\,2\,\pi^2\alpha'^2\int d^8\xi\;\; C_{(4)}\mathrm{Tr}\,[F^*_{ab}F_{ab}]\;.
\end{aligned}
$$

We now substitute the asymptotics for the background (25) and the embedding (26) in the previous formula for the action. For the instanton field we use the standard configuration (4), expressed now in the $(r,\ \alpha,\ \theta_+,\ \phi_+)$ set of coordinates. Since we are only interested in the asymptotic expansion, there is no need to introduce a self-dual configuration with respect to the full induced metric.

After these substitutions the leading term in the action, without m dependence, is zero as in AdS due to opposing contributions from the kinetic and the Wess-Zumino terms.

The first non zero contribution is second order in m:

$$
S_{D7} = -\,m^2\,\frac{T_7\,2\,(4\,\pi\alpha')^2}{3\,g_s\,g^2\,R^2}\int d^8\xi\; e^{6r}\,\frac{R^4Q^4}{(R^2\,e^{2r}+Q^2)^4}\;\cos^2\alpha\,(1+3\cos 2\alpha)\sin\theta_+ \tag{29}
$$

Thus, performing the angular integrals and writing $R\,e^r/Q = w$, we get for the potential

$$
\begin{aligned}
V(Q) &= m^2\,Q^2\;\frac{5\,T_7\,(8\,\pi^2\alpha')^2}{6\,g_s\,g^2R^4}\int_{w_0}^{\infty} dw\,\frac{w^5}{(w^2+1)^4} \\
&\simeq m^2\,Q^2\;\frac{70\,T_7\,(\pi^2\alpha')^2}{9\,g_s\,g^2\,R^4}
\end{aligned} \tag{30}
$$

assuming in the last integral $w_0 \simeq Q$.

The next correction to the potential is a constant term proportional to m^2M^2. Further corrections are negligible, being of the form of powers of $1/Q^2$ and even more suppressed by additional factors of m^2. Though this result is derived at large radius, and thus describes only the large Q sector of the field theory, the leading quadratic behaviour of the potential is a good indication of the stability of the field-theoretical vacuum.

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